Lesson 8
Futures

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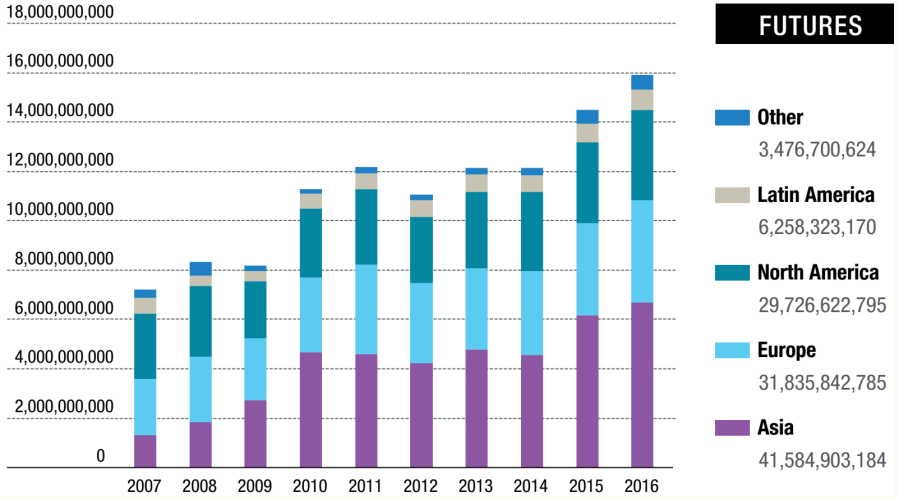
Broad Lesson Plan

Learning Outcomes

- Describe the similarities and differences of spot, forward, and futures.
- Explain the characteristics of futures contracts.
- Define and apply the important concept of contract size or contract unit.
- Distinguish the difference between daily settlement price and closing price.
- Compute the mark-to-market P&L and account balance.
- Compute the theoretical value for futures.
- Explain and apply **spot futures parity theorem**.
- Define the technical terms related to index futures.

Why Futures?



2016 Annual Volume Survey by FIA MarketVoice

What is a Futures Contract?

- Financial derivative (contract) to trade a particular commodity or financial instrument at a predetermined price at a specified time in the future.
  - **Underlying asset**: the particular commodity or financial instrument
  - **Futures market price**: the predetermined or forward price
  - **Expiration date**: the specified time in the future
- Allows market participants to offset or assume the risk of a price change of the underlying an asset over time.
- Example: Sell EUR to exchange for USD 3 months from today at the **forward price** of $1.2345.

Term Structure of Futures Prices

- **Lead** or **front month** contract: the most current contract month to expire
- **Back month** contract: further from expiration than the lead or front month contract

Month	Open	High	Low	Last	Change	Settle
18-Mar	2686.75	2691.5	2682.25	2687	-1.75	2686
18-Jun	2688.75	2693.5	2684.25A	2688.75B	-1.5	2688
18-Sep	2694.5	2696.50B	2688.50A	2692	-1.5	2691.5
18-Dec	-	2697.75B	2694.00A	2694.00A	-1.5	2693.5
19-Mar	-	-	-	-	-1.75	2696.25

CME's E-mini S&P 500 Futures Source: CME Last Updated: Friday, 22 Dec 2017 06:00 PM

What are the Characteristics of Futures Contracts?

- Standardized products created by **regulated exchanges**
- Standardized across
  - **Contract size or unit**
  - **Quality**
  - **Delivery**
    - Physical delivery procedures, time, and location
    - Cash settlement
- Traded and cleared by regulated futures exchanges
- Daily settlement price S_t
- Mark-to-market daily using daily settlement price

Examples of Regulated Futures Exchanges

- **CME Group**
- **Eurex Exchange**
- **Singapore Exchange** (SGX) Derivatives Trading
- **Japan Exchange Group** (JPX)
- **Hong Kong Exchanges and Clearing** (HKEX)
- **Taiwan Futures Exchange** (TAIFEX)
- **Korea Exchange** (KRX)
- **National Stock Exchange** (NSE)

Contract Size or Unit

- Turns the futures prices into dollars and cents upon multiplication for one contract
- Example 1: E-min S&P 500 index futures points $\times$ \$50
- Example 2: SGX Nikkei 225 index futures points $\times$ ¥500
- Example 3: Corn futures price $\times$ 5,000 bushels
- Example 4: WTI crude oil futures price $\times$ 1,000 barrels
- Example 5: Gold futures price $\times$ 100 troy ounces
- Example 6: Euro FX futures price $\times$ €125,000

Futures Expiration Month Codes

Code	Month
F	January
G	February
H	March
J	April
K	May
M	June
N	July
Q	August
U	September
V	October
X	November
Z	December

Fearsome **G**orillas **H**ave
Just **K**illed **M**any
Nyalas **Q**uietly, **U**nleashing
Very **X**enophobic **Z**eal.

Example of Quality and Delivery

Chicago SRW Wheat Futures

Contract Unit	5,000 bushels (136 Metric Tons)
Deliverable Grade	#2 Soft Red Winter at contract price, #1 Soft Red Winter at a 3 cent premium, other deliverable grades listed in Rule 14104.
Price Quotation	Cents per bushel
Tick Size	1/4 of one cent per bushel ($12.50 per contract)
Contract Months	March (H), May (K), July (N), September (U) & December (Z)
Last Trade Date	The business day prior to the 15th calendar day of the contract month.
Last Delivery Date	Second business day following the last trading day of the delivery month.

What is a Clearing House?

- An intermediary between buyers and sellers of financial instruments
- An agency or separate corporation of a futures exchange responsible for settling trading accounts, clearing trades, collecting and maintaining margin monies, regulating delivery, and reporting trading data
- Acts as third parties to all futures and options contracts, as buyers to every clearing member seller, and as sellers to every clearing member buyer.
- **Reduces counter-party risk to almost zero**.

What is Daily Settlement Price?

- A price determined by the exchange for each contract month at the end of the trading day for settlement.
- **Settlement period**: Short interval leading to the end of trading day
  - Example: between 15:14:30 and 15:15:00 Central Time (CT) for Nikkei 225 (yen) futures
- **Settlement algorithm**: Rules to calculate the settlement price from the trading activity in the settlement period for each contract month

What is Mark-to-Market?

- Going **long** is to buy; going **short** is to sell.
- Use official daily settlement price S_t set by the exchange
- Difference between S_t and your traded price P_t
  - **Long Position**: $S_t - P_t$
  - **Short Position**: $P_t - S_t$
- Your account will be credited (debited) if the difference is positive (negative).

Example of Mark-to-Market

- Buy 1 contract of Nikkei 225 index futures @ 22,900.
- Contract size is ¥500 for each index point.

Date	Futures Price	Settlement Price	Mark-to-Market P&L (Points)	Mark-to-Market P&L	Cumulative P&L	Account Balance
12-15						¥2,000,000
12-18	22,900	22,960	60	¥30,000	¥30,000	¥2,030,000
12-19		22,765	-195	-¥97,500	-¥67,500	¥1,932,500
12-20		22,825	60	¥30,000	-¥37,500	¥1,962,500
12-21		22,790	-35	-¥17,500	-¥55,000	¥1,945,000
12-22		22,830	40	¥20,000	-¥35,000	¥1,965,000

- It is as if you have sold @ the settlement price and bought it back again at the same settlement price, day after day.

Mark-to-Market P&L Calculation

- Day 12/18, buying price = 22,900; “selling price” = settlement price 22,960. So P&L = 22,960 − 22,900 = 60 index points.
- Day 12/18, “buy” the futures back @ $S_{12/18}$ = 22,960.
- Day 12/19, “buying price” = $S_{12/18}$; “selling’ price’ @ $S_{12/19}$ = 22,765. So P&L = 22,765 − 22,960 = -195 index points.
- Day 12/19, “buy” the futures back @ $S_{12/19}$ = 22,765.
- Day 12/20, ‘buying price” = $S_{12/19}$; “selling’ price’ @ $S_{12/20}$ = 22,825. So P&L = 22,825 − 22,765 = 60 index points.
- Day 12/20, “buy” the futures back @ $S_{12/20}$ = 22,825.
- etc

Concept Checker

- Instead of holding the futures to expiration, you manage to sell or liquidate your futures on Day 12/22 @ 22,800.
- Intuitively, the P&L should be 22,800 − 22,900 = -100 index points.
- Your account balance becomes ¥2,000,000 + ¥500×(-100) = ¥1,950,000.
- The mark-to-market accounting should give you the same P&L
  - Cumulative P&L up to 12/21 is 60-195+60-35 = -110 index points
  - Day 12/22, selling price = 22,800, "buying price" = $S_{12/21}$ = 22,790. So P&L = 22,800 − 22,790 = 10 index points.
  - Cumulative P&L is the same, i.e., = -110 + 10 = -100 index points.
  - In yens, ¥1,945,000 + ¥500×10 = ¥1,950,000 is also the same.

What Happens After the Last Day of Trading?

- Every contract has an expiration date T, and thus a last day of trading.
- What is the mark-to-market final settlement price?
- Final settlement price: the spot price S_T of the underlying asset
- Again, the futures exchange will use an algorithm to determine S_T.
- Question What is the P&L?
- Answer: Difference between S_T and your traded price P_t
  - **Long Position**: $S_T - P_t$
  - **Short Position**: $P_t - S_T$

Is There a Theoretical Value for Futures?

- Party A wants to buy OCBC not now but 3 months later; Party B wants to sell OCBC not now but 3 months later.
- They enter into a forward contract to trade OCBC 3 months from today at a price F_0 determined today ($t = 0$) to remove **price uncertainty**.
- S_0, the price of OCBC per share; F_0, the theoretical value
- If OCBC does not pay dividend for the next 3 months (91 days), the theoretical price F_0 is

$$F_0 = S_0 \left(1 + r_0 \times T\right), \tag{1}$$

where T is the time (year fraction) to expiration, r_0 is the **risk-free rate** for the tenor of T.

Numerical Illustration 1

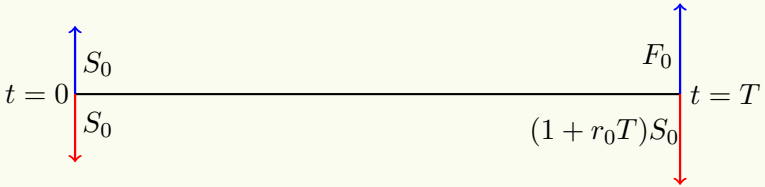
- OCBC Bank last traded price: $12.45
- 3-month SIBOR r_0: 1.40%
- Days to maturity: 91 days
- Year count convention: ACT/365
- Theoretical value

$$F_0 = \$12.45(1 + 0.0140 \times 91/365) = \$12.49.$$

- Do you want to know how the futures pricing equation (1) is arrived at?

Fair Forward Price

- **Forward contract**: futures without mark-to-market
- $t = 0$: time of forward contract initiation
- S_0: underlying asset's price at time 0
- r_0: risk-free interest rate at time 0
- F_0: the **fair forward price** of the forward contract
- $t = T$: 3 months later, the forward contract matures.



Cash Flows of Forward Seller

Self-Financing

- At time 0
  - No cash flow at the initiation of a forward contract
  - Borrow the amount S_0 at the risk-free rate of r_0
  - Buy the underlying at the price of S_0
  - Net cash flow or **net present value** of the contract is $S_0 - S_0 = 0$.

- Since the net cash flow is zero, the short position in the forward contract is said to be **self-financing**.

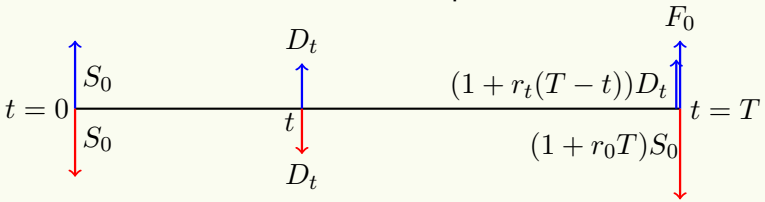
- At time T (3 months later)
  - Sell the asset for F_0 to the forward buyer
  - Return the principal plus interest $(1 + r_0 T)S_0$
  - Net cash flow = $F_0 - S_0(1 + r_0 T)$

Application of No Risk-Free Arbitrage

- If the net cash flow at time T is positive, i.e., $F_0 > S_0(1 + r_0 T)$, the forward buyer won't be happy and so won't trade because F_0 is too high.
- Conversely, if $F_0 < S_0(1 + r_0 T)$, seller is losing money because F_0 is too low and so won't trade.
- Since S_0, r_0, and F_0 are known and to be determined at time 0, the only price both the buyer and the seller are happy to trade is to have Equation (1), i.e., $F_0 = S_0(1 + r_0 T)$.
- Otherwise, no trade will occur at time 0.
- Simply, F_0 is the **forward value** of S_0.

What about Dividend?

- Suppose known at time 0, there is a dividend per share of D_t at time t.



Cash Flows of Forward Seller

- Upon receipt of D_t, invest the dividend in a risk-free instrument at the rate of r_t of tenor $T - t$.
- Cash flows at time T, by the argument of no risk-free arbitrage:

$$F_0 + \big(1 + r_t(T - t)\big)D_t = (1 + r_0 T)S_0.$$

Spot Futures Parity Theorem

- Rewrite the cash flow balance as

$$\begin{aligned} F_0 &= \big(1 + r_0 T\big) S_0 - \big(1 + r_t (T - t)\big) D_t \\ &= S_0 + \mathbf{r_0 T S_0} - \big(\mathbf{1 + r_t (T - t)}\big) \mathbf{D_t}. \end{aligned} \tag{2}$$

- **Cost** of holding the underlying stock: $r_0 T S_0$
- **Benefit** of holding the underlying stock: $\big(1 + r_t (T - t)\big) D_t$
- Equation (2) is known as the **spot futures parity theorem**.
- It is the theorem to price futures on stocks.

Numerical Illustration 2

- Using the same information in Slide 20, suppose a dividend of 30 cents per share happens on day 30.
- Suppose the 2-month risk-free rate on day 30 is 1.11%
- What's the theoretical price of the stock futures?
- Answer
  - Remaining days to expiration: $T - t = 91 - 30 = 61$
  - Theoretical price

$$F_0 = \$12.49 - \big(1 + 0.0111 \times 61/365\big) \times \$0.30 = \$12.18.$$

What is a Stock Index and Index Futures?

- Stock index: a portfolio of stocks
- Stock index futures: futures written on the stock index
- What are the examples of a stock index?
- What are the examples of a stock index futures?

What is a Free-Float Market Capitalization?

- What is free float?

  Stock i's number of shares N_i that are available to the public.
- Free float is calculated by subtracting the shares held by insiders and those deemed to be stagnant shareholders from the total number of shares outstanding.
- Stagnant holders include
  - Employee stock ownership schemes
  - Corporations not actively managing money
  - Venture capital companies
  - Government agencies
- Free-float market cap $M_{i,t}$ of component stock i of price $S_{i,t}$ at time t:

$$M_{i,t} := S_{i,t} \times N_i$$

What Does a Divisor Do?

- To compute the free-float market cap weighted index, sum up all the $M_{i,t}$, for $i = 1, 2, \ldots, n$.

$$M_t := \text{Total Free Float Market Cap} = M_{1,t} + M_{2,t} + \cdots + M_{n,t}.$$

- To convert the total market cap into an index I_t, divide by a divisor d

$$I_t = \frac{M_t}{d}.$$

- Here, divisor is a “constant” that converts the total market cap into an **index level**.
- When multiple component stocks pay dividends before futures expiration, the divisor converts the total amount into a **dividend level**.

Numerical Illustration 3

- Only 4 stocks in the index; divisor $d = 4{,}400$

	DBS	OCBC	UOB	SingTel
Price	**$20.00**	**$10.00**	**$25.00**	**$3.00**
Free-Float Shares	60,000	12,000	2,000	80,000
Market Cap	$1,200,000.00	$120,000.00	$50,000.00	$240,000.00
Days to Maturity	9	20	10	13
SIBOR	0.80%	1.20%	0.84%	0.90%
Dividend/Share	$0.40	$0.30	$1.00	$0.05
Dividend $	$24,000.00	$3,600.00	$2,000.00	$4,000.00
Dividend Level	5.46	0.82	0.45	0.91

Numerical Illustration 3 (Cont'd)

Total Market Cap: $1,610,000.00

- Index level: $1,610,000.00/4,400 = $365.91
- Suppose the futures on this 4-stock index has 30 days to maturity, and the 30-day interest rate is 1.11%.
- Interest cost: $365.91 \times 0.0111 \times 30/365 = 0.33$
- Dividend $: Dividend/Share $\times$ Free-Float Shares
- Dividend level: Equation (2)
  - DBS: $\left[\$24000 \times \left(1 + 9 \times 0.008/365\right)\right]/4400 = 5.46$
  - OCBC: $\left[\$3600 \times \left(1 + 20 \times 0.012/365\right)\right]/4400 = 0.82$
  - UOB: $\left[\$2000 \times \left(1 + 10 \times 0.0084/365\right)\right]/4400 = 0.45$
  - SingTel: $\left[\$4000 \times \left(1 + 13 \times 0.009/365\right)\right]/4400 = 0.91$
- Fair futures price: 365.91 + 0.33 - 5.46 - 0.82 - 0.45 - 0.91 = 358.60.

Summary

- Theoretical valuation of index futures

$$\begin{aligned}\text{Fair price} =& \text{Cash price}\\ &+ \text{Costs incurred to hold the index's stocks}\\ &- \text{Benefits from holding the index's stocks}\end{aligned}$$

- Insight! Futures seller holds the underlying asset on behalf of the futures buyer until maturity.
- For index futures, the cost is mainly opportunity cost, i.e., interest rate.
- For index futures, benefit includes corporate distributions: cash dividends, stock dividend, specie in kind, rights etc.