Lesson 2
Stock Market Indexes and ETFs

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Learning Outcomes

- Discuss the two standard methods to construct standard market indexes.
- Describe the similarities and differences of an index and its ETF.
- Compute the dollar amount allocated and the number of shares needed for each component stock.
- Discuss the role of a divisor.
- Compute the new divisor when a component stock has corporate actions.
- Define and describe smart beta indexes and their ETFs.

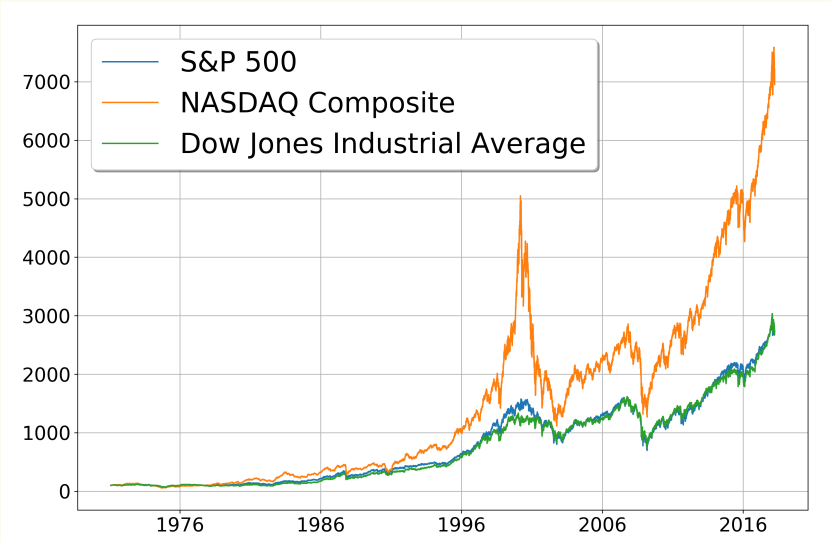
What is an index? What is an ETF?

- An **index** is a system of numbers for comparing values of interest to market participants.
- Fisher said, “For those who have made any attempt to penetrate their mysteries, index numbers seem to have a perennial fascination.”
- Today, many exchange traded funds (**ETF**) are being created to track their respective indexes as closely as possible.
- An ETF is essentially a basket of securities designed specifically to mimic the index behaviors, as closely as possible.
- Without the indexes, you would not have ETFs to invest in.

Stock Market Indexes

- Stock market index was essentially created as a **barometer** to gauge market movement or sentiment.
- Charles H. Dow, a journalist, had an idea of a stock price average to reflect the stock market condition.
- After two years of experimentation in the Customer's Afternoon Letter, the Dow Jones Transportation Index (called the "Railroad Average" in the past) made its debut on July 3, 1884, **the oldest index in history still in use**.
- Dow's indexing **algorithm**:
  1. Formulate selection criteria
  2. Select 9 railroads, a steamship company, and a telegraph company
  3. Record their end-of-day stock prices
  4. Compute the **average price** as the index (probably by hand)

Three Major Stock Market Indices



Divisor and Price-Weighted Index

Definition 2.1.

Divisor is a numerical device that gives an index provider some level of flexibility to construct, compute, re-balance, and re-constitute the index. It gives the index constructor the flexibility to maintain the index.

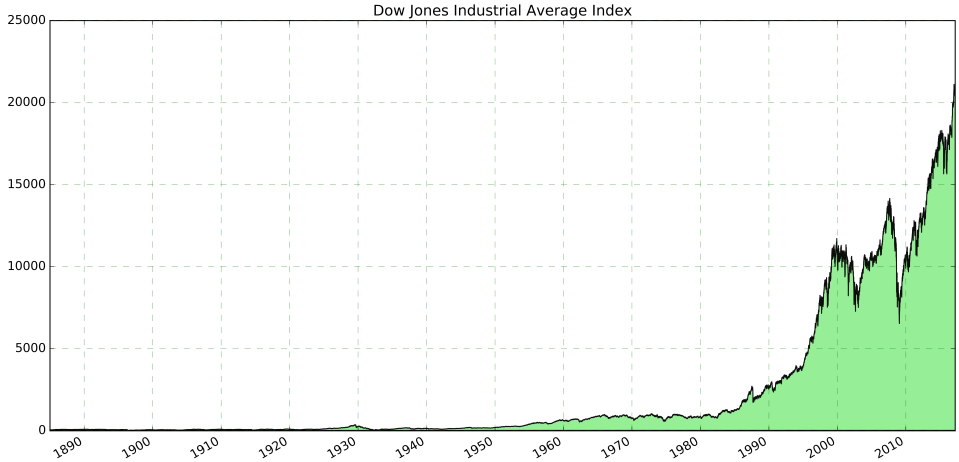
- **Dow Jones Industrial Average** The index is a 30-stock, price-weighted index that measures the performance of some of the largest US companies.
- **Dow Jones Transportation Average** The index is a 20-stock, price-weighted index that measures the performance of some of the largest US companies within the transportation industry group.
- **Dow Jones Utility Average** The index is a 15-stock, price-weighted index that measures the performance of some of the largest US companies within the utilities sector.

Dow Jones Industrial Average

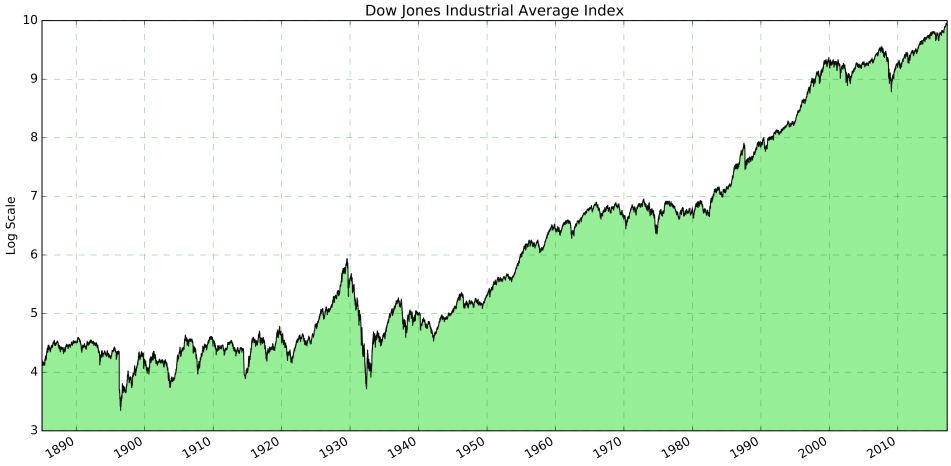
- Dow selected 12 industrial stocks and published the Industrial Average from May 26, 1896.
- At the onset of World War I, New York Stock Exchange (**NYSE**) shut down on July 31, 1914, due to large selling by foreigners (Europeans).
- On December 12, 1914, NYSE re-opened.
- On October 4, 1916, 20 **component stocks** were introduced.
- On October 1, 1928, several innovations were introduced. First, the list was expanded from 20 to 30. Second, a divisor was introduced. Third, Dow Jones began calculating the high-low-close for the component stocks.

Reference: **Global Financial Data**

Historical Plot of DJIA Index



Historical Plot of DJIA Index in Log Scale



SYMBOL	NAME	LAST
AEP-Q	American Electric Power Company	73.35
AES-N	The Aes Corp	14.00
ATO-N	Atmos Energy Corp	107.79
AWK-N	American Water Works	115.5
D-N	Dominion Energy Inc	39.53
DUK-N	Duke Energy Corp	86.33
ED-N	Consolidated Edison Company	86.59
EIX-N	Edison International	61.84
EXC-Q	Exelon Corp	38.40
FE-N	Firstenergy Corp	34.99
NEE-N	Nextera Energy	51.52
PEG-N	Public Service Enterprise Group Inc	59.16
SO-N	Southern Company	65.43
SRE-N	Sempra	68.14
XEL-Q	XCEL Energy Inc	58.05

- Date: 2013-10-23
- Sum of last prices: 960.62
- Divisor: 1.1978034655451
- **Dow Jones Utility Index**: 801.98

Price Weighted Method

- Suppose the number of component stocks is N.
- At time t, the prices of component stocks are $P_{i,t}$, $i = 1, 2, \ldots, N$.
- Sum up all the prices to obtain the total price P_t:

$$P_t = \sum_{i=1}^{N} P_{i,t}.$$

- Given current total price P_t, the current index I_t is calculated as

$$I_t = \frac{P_t}{P_{t-1}} I_{t-1}.$$

- It can be rewritten as $I_t = \frac{P_t}{\frac{P_{t-1}}{I_{t-1}}}$, and $D := \frac{P_{t-1}}{I_{t-1}} = \frac{P_t}{I_t}$ is the **divisor** for the index.

How to Construct a Price-Weighted Index?

- Example

Company	3M Co.	Boeing Co.	Intel Corp.	Pfizer Inc.
Last Price	$241.14	$320.26	$42.50	$36.47

- What is the index level at inception?
- What is the divisor at inception?

ETF on Price-Weighted Index

- Equal number of shares for every component stock!
- One **round lot** is usually 100 shares.

Company	3M Co.	Boeing Co.	Intel Corp.	Pfizer Inc.
Last Price	$241.14	$320.26	$42.50	$36.47
Shares	100	100	100	100
Cost	$24,114.00	$32,026.00	$4,250.00	$3,647.00
Fees	$4.95	$4.95	$4.95	$4.95
Total	$24,118.95	$32,030.95	$4,254.95	$3,651.95

- You must have at least $64,056.80 to construct the ETF.

Nikkei 225 Index

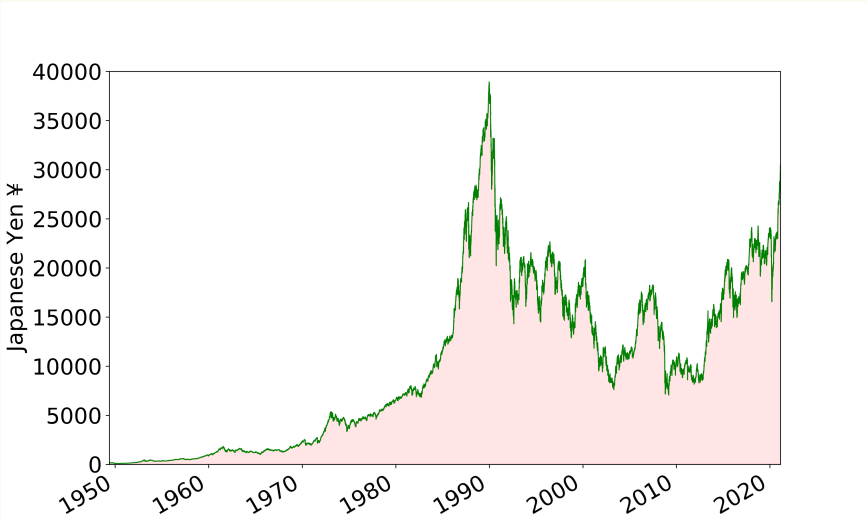
Definition 2.2.

An **adjusted-price-weighted index** of n component stocks is formally computed as, given all the last traded prices $P_{i,t}$ and the **price adjustment factor** c_{i,s_i} of the component stocks at a given time t,

$$I_t = \frac{\sum_{i=1}^{n} c_{i,s_i} P_{i,t}}{d_s}.$$

The **divisor** d_s is last updated at time $s \leqq t$. Also, $s_i \leqq t$ for $i = 1, 2, \ldots, n$, which is the time when the price adjustment factor of stock i is last updated.

Nikkei 225 Index



Overcoming the Weakness of Stock Split

- In a **stock split** of **split factor** f, the price will be adjusted on the effective day t:

$$P_{i,t} \longrightarrow \frac{P_{i,t}}{f} =: \check{P}_{i,t}.$$

- In the usual **stock split** by which more shares are created, $f > 1$. On the hand, in the case of a **reverse stock split**, $f < 1$.
- The split factor f can be absorbed by adjusting the **price adjustment factor** c_{i,s_i} without changing the divisor as follows:

$$c_{i,s_i} \longrightarrow c_{i,s_i} f =: c_{i,t_i}.$$

- This is because

$$c_{i,s_i} P_{i,t} = c_{i,s_i} \frac{f}{f} P_{i,t} = c_{i,s_i} f \frac{P_{i,t}}{f} = c_{i,t_i} \check{P}_{i,t}.$$

Evidence

Regarding the Stock Split of NTT in Nikkei 225

June 16, 2023
Nikkei Inc.

NIPPON TELEGRAPH AND TELEPHONE CORPORATION (NTT, 9432), a Nikkei Stock Average (**Nikkei 225**) constituent, is planning a stock split of 1 to 25. From the market open of the ex-right date, which is June 29, Nikkei Inc. will change the Price Adjustment Factor of NTT for Nikkei 225 from 0.4 to 10.

Since the price level of NTT in the Nikkei 225 will stay the same, the divisor will not be changed by this event.

ETF of Price-Weighted Index

- Suppose we buy 100 shares for each of the constituent stocks of Dow Jones Utility Average (DJUA) index. Before transaction costs, from Slide 11, the amount of money needed is \$960.62 $\times$ 100 = \$9,606.20. It is easy to see that the current value of our ETF is 100 times of

$$I_t d_s = \sum_{i=1}^{n} P_{i,t} = \$960.62.$$

- In the case of Nikkei 225 index, because of the price adjustment factors, the equal-share approach does not work anymore.

- In principle, we buy each share according to the **adjustment factor** c_{i,s_i}. Put differently, the price adjustment factor may be interpreted as the number of shares.

Market Capitalization

Definition 3.1.

Market capitalization of a company at time t, denoted by M_t, is defined as

$$M_t := N_s P_t.$$

Here P_t is the price of the stock per share at time t, and N_s is the number of outstanding shares issued by the company, correct as at time s, which of course is prior to time t.

- Suppose t is the **ex date** of a stock split characterized by an **split factor** f.
- As a result of the stock split, the number of shares becomes $S_t := fS_s$ and the share price becomes $\check{P}_t := P_t/f$.
- It turns out that

$$M_t = S_s P_t \frac{f}{f} = fS_s \times \frac{P_t}{f} = S_t \check{P}_t.$$

Market-Cap Weighted Index

Definition 3.2.

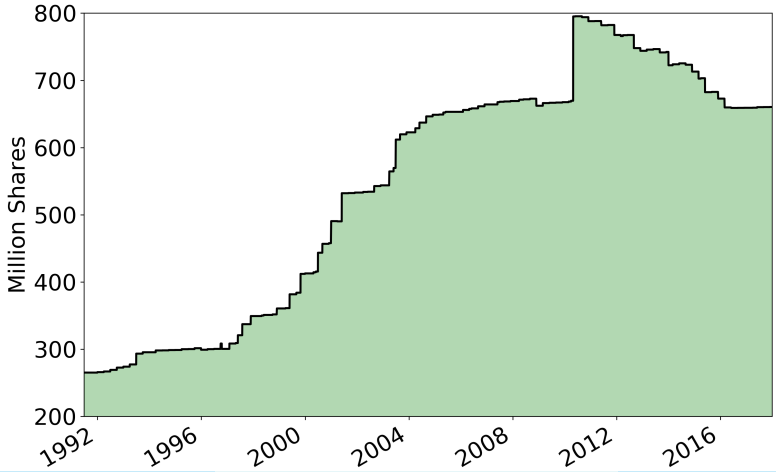
Let the **market capitalization** of a **constituent stock** be $M_{i,t}$. The index based on the market capitalization is then defined as

$$J_t := \frac{\sum_{i=1}^{n} M_{i,t}}{d_s} = \frac{\sum_{i=1}^{n} S_{s_i} P_{i,t}}{d_s}.$$

The resulting index is referred to as the **market-cap weighted index** or **value-weighted index**. The **divisor** is last updated at time $s \leqq t$, and S_{s_i} is the last updated number of shares for stock i.

- The number of shares S_{s_i} issued by every company does not remain unchanged.

Outstanding shares of AES Corporation (Data source: CRSP).



General Index Formula

Proposition 1

If the **divisor** remains unchanged from time t to time u, then

$$J_u = J_t \times \frac{\sum_{i=1}^{n} S_{s_i} P_{i,u}}{\sum_{i=1}^{n} S_{s_i} P_{i,t}}. \tag{1}$$

Proof.

By definition, at time t, $d_s = \frac{\sum_{i=1}^{n} S_{s_i} P_{i,t}}{J_t}$. Likewise, at time u, $d_s = \frac{\sum_{i=1}^{n} S_{s_i} P_{i,u}}{J_u}$. Equating these two expressions, we obtain (1). □

Real-World VICE Index and ETF

Ticker	V	ISRG	SIRI	EA
Name	Visa	Intuitive Surgical	Sirius XM	Electronic Arts
Last Traded	$129.93	$458.79	$6.97	$132.00
Shares Outstanding	1,786,164,000	112,299,000	4,491,864,000	306,728,000
Market Capitalization	$232,076,288,520	$51,521,658,210	$31,308,292,080	$40,488,096,000
Weight	65.30%	14.50%	8.81%	11.39%
Funds Allocated	$65,301.07	$14,497.04	$8,809.45	$11,392.44
Tentative Shares	502.59	31.60	1,263.91	86.31
Rounded Shares	503	31	1,264	86
Actual Funds Spent	$65,354.79	$14,222.49	$8,810.08	$11,352.00

- Weight is calculated as follows.

$$w_i := \frac{S_{s_i} P_{i,t}}{\sum_{j=1}^{n} S_{s_j} P_{j,t}}. \tag{2}$$

VICE Index

- The total **market capitalization**, $355,394,334,810, is the sum of the market values of the four VICE stocks. That is,

$$\$232,076,288,520 + \$51,521,658,210 + \$31,308,292,080 + \$40,488,096,000 = \$355,394,334,810.$$

- To begin the VICE index at the level of 100, the initial divisor is set at

$$\frac{\$355,394,334,810}{100} = 3,553,943,348.10.$$

We can then proceed to compute the weight of each stock with respect to the total market capitalization.

VICE ETF

- To construct an ETF based on the VICE index, we need to allocate our fund, say $100,000, among the four VICE stocks. The weights we have computed earlier allow us to do so.
- We ration the fund to each stock by multiplying $100,000 with its weight.
- To obtain the tentative numbers of shares to buy, divide the funds by the respective last traded share prices.
- Finally, we need to exercise discretion to either round up or round down the calculated numbers, because the smallest unit of trading is one share.
- A key consideration here is that the actual total funds spent should be very close to, yet not exceeding the budget of $100,000.

Free Float

Definition 3.3.

The notion of **free float** refers to the issued shares of a company that are in the hands of public investors, as opposed to shares closely held by investors who have agenda other than just an investment in the company.

- The introduction of free float is a major improvement to the industry.
- **Outstanding shares** are all shares issued by a company.
- **Restricted shares** are those shares held by company founders, officers, and directors; government, long-term shareholders, trusts, foundations, and so on.
- So, Free float $=$ Outstanding Shares $-$ Restricted Shares

Examples of Market Cap Weighted Indexes

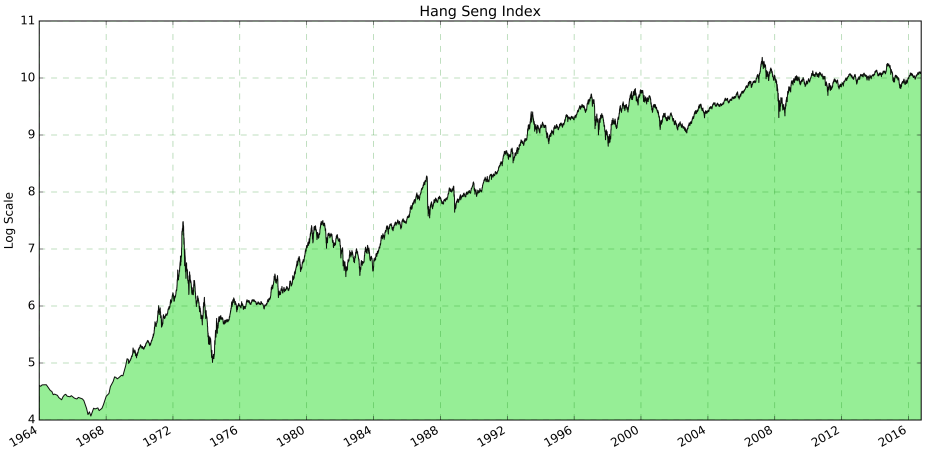
- Most stock market indexes are free-float market cap weighted.
- S&P 500 Index
- **S&P Dow Jones Indices**
- Topix Index
- Hang Seng Index
- **MSCI** indexes
- **FTSE-Russell indexes**

Hang Seng Index: A Case Study

Source: http://www.aastocks.com/en/stocks/quote/dynamic-chart.aspx?

Historical Hang Seng Index

Historical Hang Seng Index in Log Scale



Index Calculation Formula

Source: https://www.hsi.com.hk/static/uploads/contents/en/dl_centre/methodologies/index_methodology_guide_e.pdf

$$I_t = \frac{\sum_{i=1}^{N} P_{i,t} S_{s_i} f_i c_i a_i}{d_s}$$

- N : number of component stocks
- P_t : is current price at time t
- P_{t-1} : losing price at time $t-1$
- S_{s_i} : outstanding or issued shares
- f_i : **free float** adjustment factor, between 0 and 1
- c_i : capping factor
- a_i : adjustment factor

Data Sources

Bloomberg Ticker	1 HK Equity	2 HK Equity	3 HK Equity	4 HK Equity	5 HK Equity
Company Name	CKH Holdings	CLP Holdings	HK & China Gas	Wharf Holdings	HSBC Holdings
Cap Factor	1	1	1	1	0.6155
LAST_PRICE	97.10	81.60	15.46	65.75	66.50
Free Float Adjustment Factor	0.7	0.75	0.6	0.4	1
EQY_FREE_FLOAT_PCT	67.60978	71.41111	58.19519	38.89861	99.97364
Shares Issued	3,857,678,500	2,526,450,570	12,717,042,258	3,033,227,327	20,378,023,222
BS_SH_OUT	3,857.678500	2,526.450570	12,717.042258	3,032.000000	20,191.586214

- Company Name, Cap Factor, and Free Float Adjustment Factor are obtained from the **Hang Seng Indexes Company**.
- Shares Issued is obtained from the **Hong Kong Exchange**.
- The rest (in upper case) are from Bloomberg.

Remarks on Standard Indexes

- Based on prices directly
- Self-rebalancing
- Rebalancing due to corporate actions
- **Stock split** of a component stock: price-weighted index must change the divisor; market cap weighted index need not change.
- **Shares buy-back** and **recapitalization** of a component stock: price-weighted index is not affected; market cap weighted index must change the divisor.

How to Change the Divisor?

- **The basic principle is that the index value should not change when adjusting the divisor after the market has closed.**
- For price-weighted index I_t, suppose Stock 1 has a stock split.

$$I_t = \frac{\sum_{i=1}^{N} P_{i,t}}{D_{\text{old}}} = \frac{P'_{1,t} + \sum_{i=2}^{N} P_{i,s_i}}{D_{\text{new}}}.$$

- For free float market cap weighted index J_t, suppose stock 1 has a recapitalization: S_{1,s_i} becomes $S'_{1,t}$. Then

$$J_t = \frac{\sum_{i=1}^{N} P_{i,t} S_{i,s_i}}{D_{\text{old}}} = \frac{P_{1,t} S'_{1,t} + \sum_{i=2}^{N} P_{i,t} S_{i,s_i}}{D_{\text{new}}}.$$

Overview of Alternatively Weighted Indexes

- Alternative weighted indexes are not based on prices directly.
- They are designed to reflect the performance of investment strategies with specific diversification, risk reduction, or factor objectives.
- Alternative weighting schemes include
  - Equal
  - Fundamental
  - Minimum variance
  - Factor

What is equal weight?

Definition 4.1.

Equally weighted index, also known as **unweighted index**, is defined as the index constituted by giving **equal dollar amount** and thus **equal weight** to every component stock.

- Equally weighted index can be easily constructed by averaging all simple returns of the component stocks.
- At any given time, it is a **cross-sectional average** across all securities in the basket.
- If we chronologically collect the averages of the simple returns, we can easily construct an equally weighted index.

Example of Geometric Average

Example 4.2.

- Suppose the cross-sectional averages of simple returns at Days 1 to 5 are, respectively, 1%, -2%, 3%, 10%, and -5%.
- We can start at Day 0 with a base index value of 100.
- On Day 1, the index value becomes $100 \times (1 + 0.01) = 101$.
- Likewise on Day 2, we have $101 \times (1 - 0.02) = 98.98$.
- On Day 3, the index value becomes 101.95, and 112.15 on Day 4.
- Finally, on Day 5, we obtain 106.54.

Value Line Index

- As a practical example of an unweighted index, consider the **Value Line Geometric Composite Index**.
- It was introduced on June 30, 1961.
- This market benchmark is an equally weighted price index of all stocks covered in **The Value Line Investment Survey**.
- Geometric refers to the averaging technique used to compute the average.

$$V_t = \left(\frac{P_{1,t}}{P_{1,t-1}} \frac{P_{2,t}}{P_{2,t-1}} \cdots \frac{P_{N,t}}{P_{N,t-1}} \right)^{\frac{1}{N}} = \left(\left(1 + R_{1,t}\right)\left(1 + R_{2,t}\right) \cdots \left(1 + R_{N,t}\right) \right)^{\frac{1}{N}}$$

$$=: (1 + g) V_{t-1}$$

How to Create an Equally Weighted ETF?

Example 4.3.

- The fund of $100,000 is divided equally among the four stocks. The rounded numbers of shares are tabulated as follows:

Ticker	V	ISRG	SIRI	EA
Name	Visa	Intuitive Surgical	Sirius XM	Electronic Arts
Last Traded	$129.93	$458.79	$6.97	$132.00
Funds Allocated	$25,000.00	$25,000.00	$25,000.00	$25,000.00
Tentative Shares	192.41	54.49	3,586.80	189.39
Rounded Shares	193	54	3,587	189
Actual Funds Spent	$25,076.49	$24,774.66	$25,001.39	$24,948.00

- The total actual money spent is $99,800.54, giving rise to a cash balance of $199.46.
- This amount should be sufficient to pay for the transaction costs of acquiring the rounded shares.

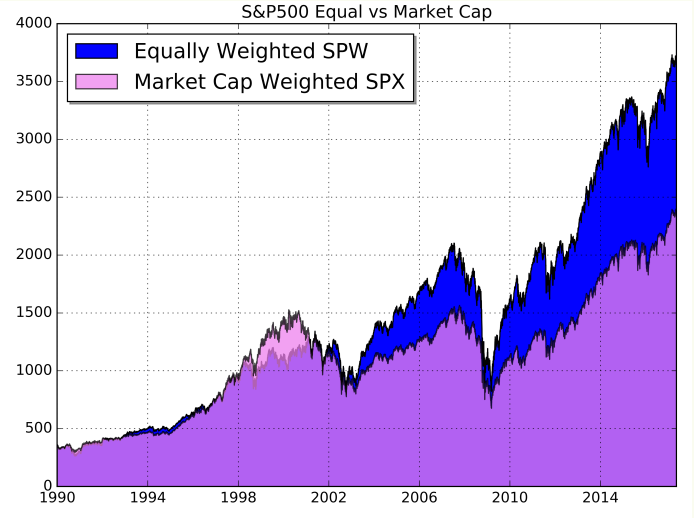
NAV of ETF

Definition 4.4.

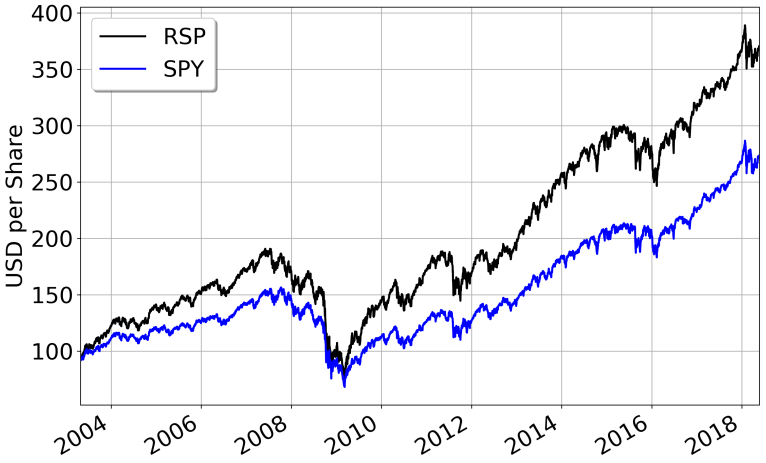
NAV is the market value of a mutual fund's or ETF's total assets, minus liabilities, divided by the number of shares outstanding.

- The value of each asset is determined by the midpoint between the bid-offer prices as of the closing time of the stock exchange on business days.
- The exchange-traded price P_t of an ETF usually are not equal to its NAV N_t.
- Define the difference
$$P_t - N_t$$
- If the difference is positive (negative), we say that the ETF is selling at a **premium** (**discount**).

Equally Weighted vs Market Cap Weighted



Value-Weighted versus Unweighted ETFs



Research Topics

- Why does RSP almost always outperform SPY?
- Co-integration test of RSP and SPY
- Optimal re-balancing frequency of unweighted portfolio
- Define the spread RSP − SPY and model its dynamics.
- Is the widening and narrowing spread a signal for the market?

Fundamentally Weighted Index

- A fundamentally weighted index is constructed by calculating the economic size of each company within the index's universe, based on such factors as: revenues; cash flow; book value; and dividends. The index is then weighted to reflect the relative economic size of each stock to the overall universe.

- Examples
  - Last Year's Earnings of Stocks, A, B, and C are, respectively, $900 million, $600 million, and $300 million.
  - So the weights are, respectively, 50%, 33.33%, and 16.67%.

- Since the index weights are not based on market cap, it is not influenced by short term elements.

Reference: http://valueweightedindex.com/

Fundamental Indexation

Reading: **Robert D. Arnott, Jason Hsu, and Philip Moore, Financial Analysts Journal 61, 83–99 (2005)**

Table: Performances of Fundamentally Weighted Indexes, 1962-2004

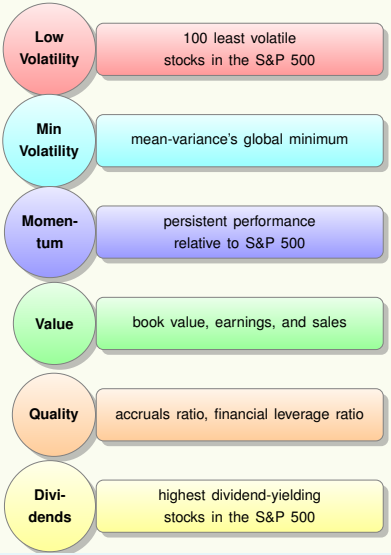
Portfolio/ Index	Ending Value of $1	Geometric Return	Volatility	Sharpe Ratio	Excess Return	Tracking Error	Information Ratio	t-Statistic for Excess Return
Reference	$68.95	10.35%	15.2%	0.301	–	–	–	–
S&P 500	$73.98	10.53%	15.1%	0.315	0.18%	1.52%	0.12	0.76
Book	$136.22	12.11%	14.9%	0.426	1.76%	3.54%	0.50	3.22
Income	$165.21	12.61%	14.9%	0.459	2.26%	3.94%	0.57	3.72
Revenue	$182.05	12.87%	15.9%	0.448	2.52%	5.03%	0.50	3.25
Sales	$184.95	12.91%	15.8%	0.452	2.56%	4.93%	0.52	3.36
Dividends	$131.37	12.01%	13.6%	0.458	1.66%	5.33%	0.31	2.02
Employment	$156.83	12.48%	15.9%	0.423	2.13%	4.64%	0.46	2.98
Average	$159.44	12.50%	15.2%	0.444	2.15%	4.57%	0.47	3.09
Composite	$156.54	12.47%	14.7%	0.455	2.12%	4.21%	0.50	3.26

The Reference portfolio is a market cap-weighted index of 1,000 component stocks.

Smart Beta

- The origin of alternatively weighted indexes is the idea of “smart beta”.
- A definition of **smart beta**
  Smart beta is a type of transparent investment strategies that are based on **rules** designed to **provide** specific exposure to certain risk factors or/and market segments.
- In implementing a smart beta investment strategy, you would systematically select, weigh, and re-balance portfolio holdings on the basis of factors or characteristics other than market capitalization.
- The equity factors that appear to be most robust over time and across countries are company size, value, low volatility, momentum, quality, dividends, and share buybacks.

S&P 500 Smart Beta Indices



Summary Statistics of S&P 500 Smart Beta

Sample Period: Dec 31, 2001 through Dec 31, 2016. Reference: **Factoring in Factors, S&P Dow Jones Indices**

	Equal Weight	Enhanced Value	Low Volatility	Momentum	Quality	Dividend Aristocrats	Buyback	S&P 500
Performance								
Return (%)	9.29	7.82	9.13	7.21	9.21	9.71	10.90	6.69
Volatility (%)	16.97	19.65	10.30	14.11	13.55	12.90	16.00	14.35
Sharpe Ratio	0.47	0.33	0.77	0.42	0.59	0.66	0.60	0.38
Sortino Ratio	0.83	0.57	1.34	0.74	1.04	1.17	1.04	0.67
Performance Relative to S&P 500								
Monthly Alpha (%)	0.14	(0.00)	0.39	0.13	0.24	0.32	0.32	
Beta to Market	1.15	1.26	0.61	0.85	0.92	0.83	1.05	
Beta to Up Market	1.20	1.22	0.51	0.69	0.96	0.93	1.07	
Beta to Down Market	1.13	1.38	0.71	0.81	0.92	0.83	1.09	
Correlation	0.97	0.92	0.85	0.86	0.97	0.92	0.94	
Extreme Risk Statistics								
Best Monthly (%)	18.7	22.0	6.8	11.5	10.5	12.2	16.3	10.9
Worst Monthly (%)	(21.1)	(23.5)	(12.8)	(14.0)	(16.3)	(13.2)	(21.2)	(16.8)
+ ve Monthly (%)	62.8	61.7	62.8	64.4	65.0	64.4	63.3	64.4
Drawdown History								
Max Drawdown (%)	(54.9)	(67.9)	(35.4)	(44.0)	(44.4)	(44.1)	(53.0)	(50.9)
Peak Date	07-May	07-May	07-May	07-Oct	07-Oct	07-May	07-May	07-Oct
Trough Date	09-Feb	09-Feb	09-Feb	09-Feb	09-Feb	09-Feb	09-Feb	09-Feb
Peak to Trough	21	21	21	16	16	21	21	16
Recovery Length	23	50	22	37	24	20	23	37

Stock Splits and Price-Weighted Index

- Without loss of generality, suppose stock 1 has a **stock split** at time t.
- Using the pre-split price $P_{1,t}$, the index is computed as

$$I_t = \frac{1}{d_s}\left(P_{1,t} + \sum_{i=2}^{n} P_{i,t}\right).$$

- We have isolated $P_{1,t}$ so as to see the change clearly. Now, $\check{P}_{1,t} = P_{i,t}/f$ at the close of the trading session, where $f > 1$ for a stock split.
- To obtain the same I_t, we need to find a new divisor d_t so that

$$I_t = \frac{1}{d_t}\left(\check{P}_{1,t} + \sum_{i=2}^{n} P_{i,t}\right).$$

Stock Splits and Price-Weighted Index (つづき)

- Solving for the new divisor d_t, we obtain

$$d_t = \frac{\check{P}_{1,t} + \sum_{i=2}^{n} P_{i,t}}{I_t} = \left(\check{P}_{1,t} + \sum_{i=2}^{n} P_{i,t} \right) \times \frac{1}{I_t} = d_s \times \frac{\check{P}_{1,t} + \sum_{i=2}^{n} P_{i,t}}{P_{1,t} + \sum_{i=2}^{n} P_{i,t}}. \tag{3}$$

Stock Split and Divisor Change

- The new divisor will be smaller than the old divisor d_s, as can be seen from (3), which is rewritten as, with $c := \sum_{i=2}^{n} P_{i,t}$,

$$\frac{d_t}{d_s} = \frac{\frac{P_{1,t}}{f} + c}{P_{1,t} + c} < 1.$$

- Conversely, for a **reverse stock split**, $d_t > d_s$.

Value-Weighted Index and Change in Free Float

- Without loss of generality, suppose the number of issued shares of Stock 1 increases by $\Delta S_{1,t}$, so that its **market capitalization** becomes

$$P_{1,t}S_{1,s_i} \longrightarrow P_{1,t}S_{1,s_i} + P_{1,t} \times \Delta S_{1,t}.$$

- To account for the additional market capitalization, we need to adjust the **divisor** to

$$d_t = \frac{P_{1,t}S_{1,s_i} + P_{1,t} \times \Delta S_{1,t} + \sum_{j=2}^{n} P_{j,t}S_{j,s_j}}{J_t} = d_s \times \frac{P_{1,t} \times \Delta S_{1,t} + \sum_{j=1}^{n} P_{j,t}S_{j,s_j}}{\sum_{j=1}^{n} P_{j,t}S_{j,s_j}}$$

$$= d_s \left(1 + \frac{P_{1,t} \times \Delta S_{1,t}}{J_t}\right). \tag{4}$$

Change in Free Float and Divisor Change

- From (4), we see that the new **divisor** d_t will be larger than the old divisor d_s by a fraction
$$\frac{P_{1,t} \times \Delta S_{1,t}}{J_t}.$$
- Conversely, when a **share repurchase** occurs, $\Delta S_{1,t}$ is negative and thus the new divisor will be smaller than the old divisor.
- Given that the **corporate actions** of issuance of more shares and share buybacks occur more frequently than stock splits, it seems that updating of divisor for value-weighted index has to be carried out more often than for the price-weighted index.

The Case of Unweighted Index

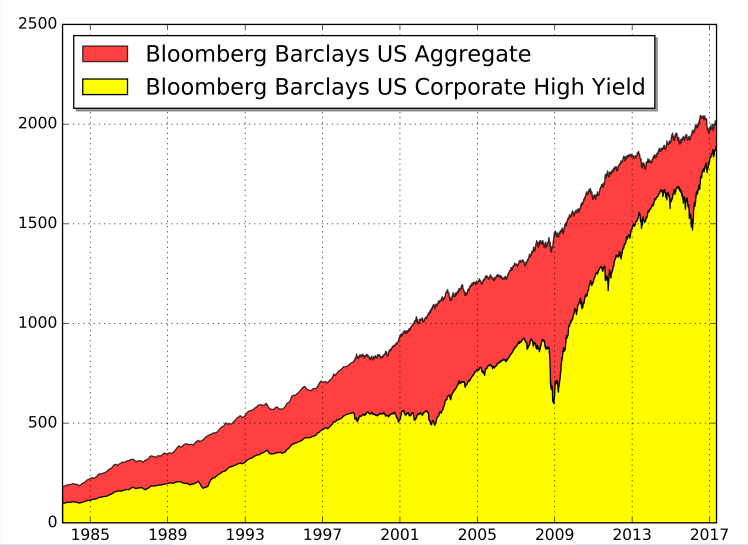
- The **divisor** of **unweighted index** is constant.
- The crux of **equal weighting** is the computation of every constituent stock return.
- If a stock undergoes a stock split, we have to use the split-adjusted price to compute the return.
- Re-balancing of unweighted index is to sell some shares of the winning stock and at the same time use the sales proceeds to buy more shares of the losing stock, so that the capital in dollar is equally divided again.
- So unweighted index is a strategy betting on (relative) **price reversal**.

Summary of Re-balancing

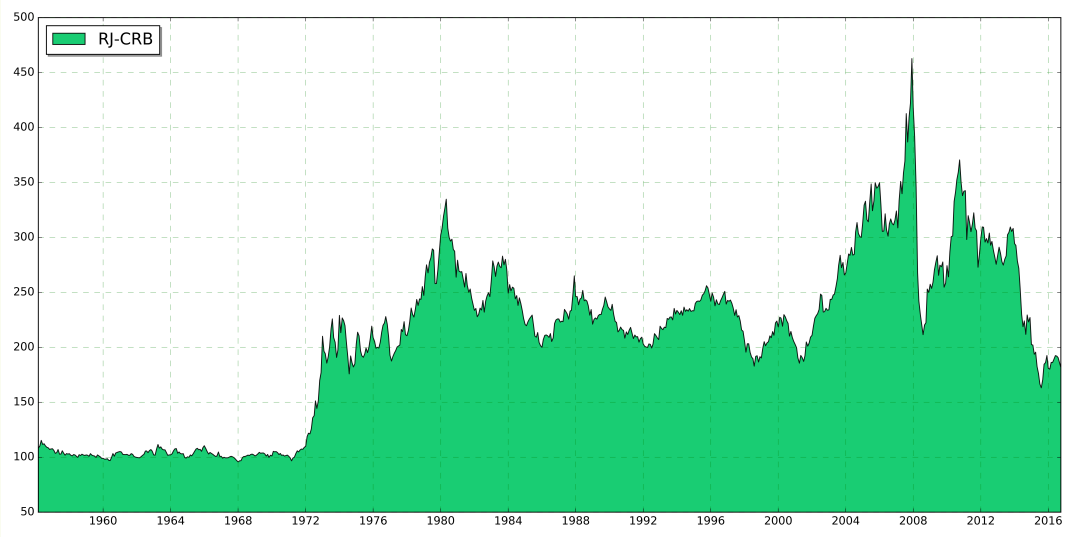
- Does the **divisor** need to change?

Method	Stock Split	Change in Issued Shares
Price-weighted	Yes	No
Value-weighted	No	Yes
Equally weighted	No	No

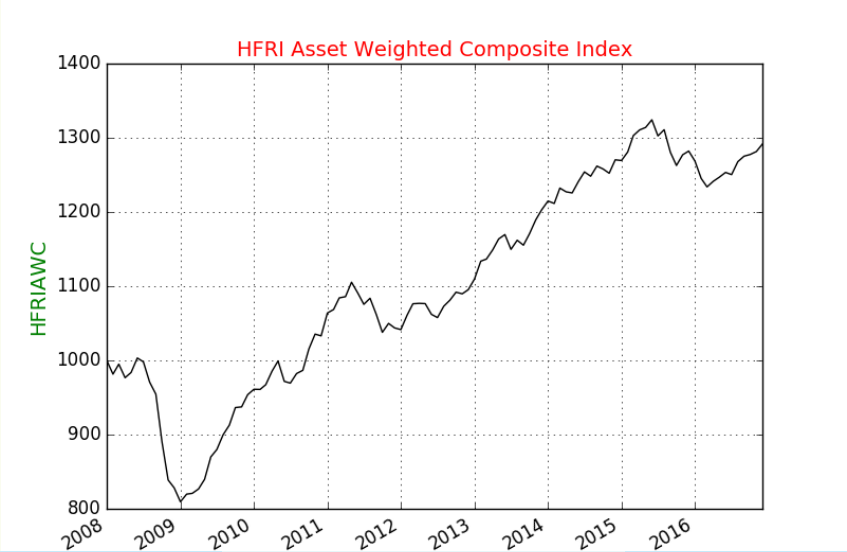
Bond Indices



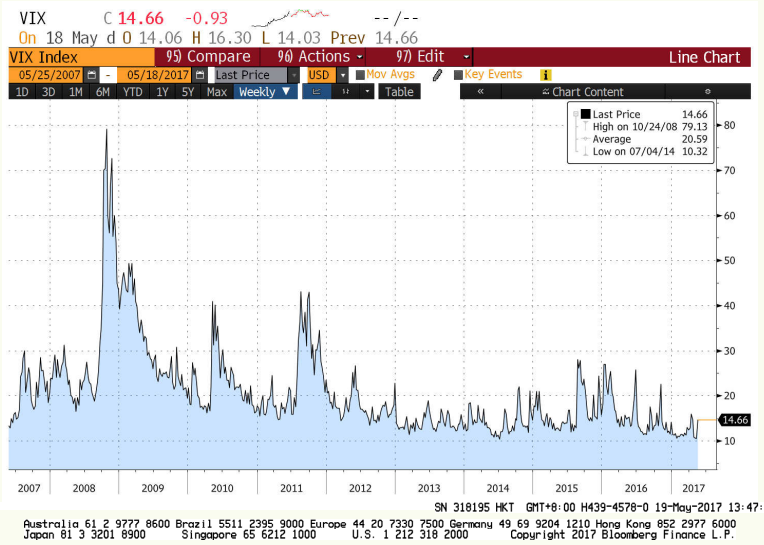
Reuters/Jefferies-CRB Index



Hedge Fund Index



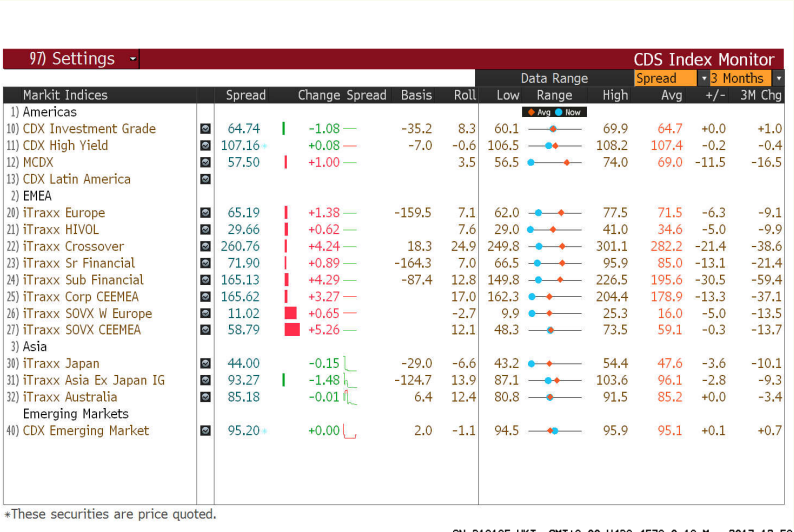
Volatility Index (VIX)



Deutsche Bank Index Quant's Credit Index

DBIQ is responsible for launching, daily calculation, **re-balancing**, and retiring of proprietary Deutsche Bank benchmarks.

Markit Indices



97) Settings — CDS Index Monitor

Markit Indices		Spread	Change Spread	Basis	Roll	Data Range Low	Range	High	Spread Avg	Spread +/-	Spread 3M Chg
1) Americas							Avg Now				
10) CDX Investment Grade		64.74	-1.08	-35.2	8.3	60.1		69.9	64.7	+0.0	+1.0
11) CDX High Yield		107.16	+0.08	-7.0	-0.6	106.5		108.2	107.4	-0.2	-0.4
12) MCDX		57.50	+1.00		3.5	56.5		74.0	69.0	-11.5	-16.5
13) CDX Latin America											
2) EMEA											
20) iTraxx Europe		65.19	+1.38	-159.5	7.1	62.0		77.5	71.5	-6.3	-9.1
21) iTraxx HIVOL		29.66	+0.62		7.6	29.0		41.0	34.6	-5.0	-9.9
22) iTraxx Crossover		260.76	+4.24	18.3	24.9	249.8		301.1	282.2	-21.4	-38.6
23) iTraxx Sr Financial		71.90	+0.89	-164.3	7.0	66.5		95.9	85.0	-13.1	-21.4
24) iTraxx Sub Financial		165.13	+4.29	-87.4	12.8	149.8		226.5	195.6	-30.5	-59.4
25) iTraxx Corp CEEMEA		165.62	+3.27		17.0	162.3		204.4	178.9	-13.3	-37.1
26) iTraxx SOVX W Europe		11.02	+0.65		-2.7	9.9		25.3	16.0	-5.0	-13.5
27) iTraxx SOVX CEEMEA		58.79	+5.26		12.1	48.3		73.5	59.1	-0.3	-13.7
3) Asia											
30) iTraxx Japan		44.00	-0.15	-29.0	-6.6	43.2		54.4	47.6	-3.6	-10.1
31) iTraxx Asia Ex Japan IG		93.27	-1.48	-124.7	13.9	87.1		103.6	96.1	-2.8	-9.3
32) iTraxx Australia		85.18	-0.01	6.4	12.4	80.8		91.5	85.2	+0.0	-3.4
Emerging Markets											
40) CDX Emerging Market		95.20	+0.00	2.0	-1.1	94.5		95.9	95.1	+0.1	+0.7

*These securities are price quoted.

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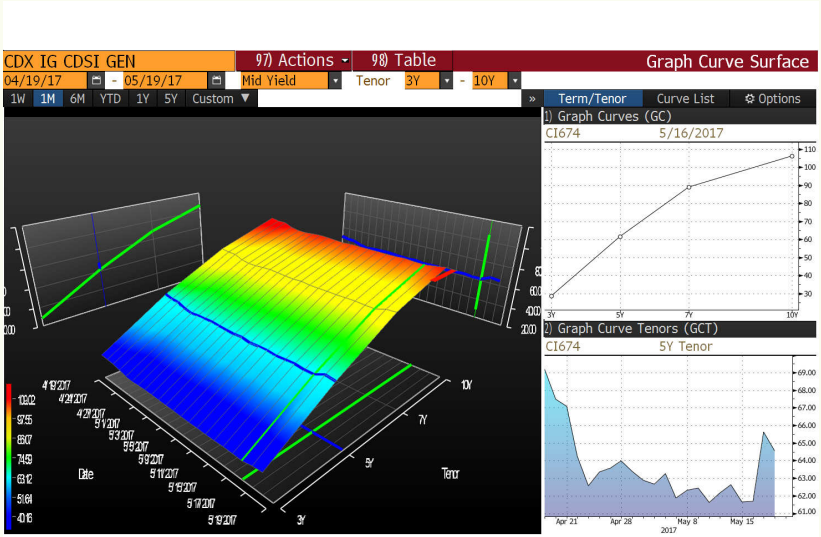
Investment Grade



High Yield


CDX HY CDSI GEN 5Y PRC 107.115 +0.111 107.081/107.149
At 18 May
Source CBBT
CDX HY CDSI GEN 5Y PRC Corp
Page 1/2 Description: CDS Index
94) Notes
95) Buy
96) Sell
97) Settings
Pages
1) Index Info
2) Roll Info
Index Information
Markit CDX North America High Yield Index is composed of 100 non-investment grade entities, distributed among 2 sub-indices: B, BB. All entities are domiciled in North America. Markit CDX indices roll every 6 months in March & September.
Contract Information
Currency USD Day Cnt ACT/360
Tenor 5Y Cpn Freq Q
Dsc Curv US Fixing Swap Curve
Region America
ISDA Definitions Year 2014
Cpn (bps) 500 Factor 1.0
Recovery 0.30 Version 1
Quote Type Price Series 28
Start Date 03/20/17 Start Cnst 100
Mty Date 06/20/22
Restruct No Restructuring
Identifiers
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Full Name CDX HY CDSI GEN 5Y PR
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Takeaways

- Index weighing methods are investment strategies.
- Both standard and alternative equity market indexes are paper portfolios that assume zero market friction.
  1. Stock trading is ideally liquid; no price impact in trading
  2. Number of shares can be in decimals; arbitrary positive real numbers
  3. No commission and fees in construction, re-balancing, and reconstitution
- ETF of an equity index has market friction
- Most smart beta indexes seem to be superior than the S&P 500 index in terms of Sharpe ratio.
- Many indexes for other asset classes