Preface

As much as a point in space is represented by a tuple of 3 real numbers, all the geometrical properties of a spatial diagram can be expressed as a formula of real variables. Accordingly, if one has the knowledge of algebra and analysis, then the properties of the diagram can be understood in a logically correct manner. But to develop the geometry further, it is insufficient to have only the knowledge of algebra and analysis; one needs to have an intuitive grasp of the diagrams. The same applies to probability theory.

Modern probability theory, when described by the language of measure theory, logically becomes an area of analysis entirely. But to truly enjoy probability theory, one must, from the background of intuitive understanding of the probabilistic phenomena, seek the direction of development for the probability theory.

The first chapter of this book focuses on the simplest finite trials, and explains the methods that analytically describe the probabilistic phenomena. The subsequent chapters develop the generic probability theory.

With respect to measure theory, many excellent books have already been published. Even in the current series[1], there is one item on that. That said, the examples and various properties of measure theory are summarized in Chapter 2 for references in the subsequent chapters.

In Chapter 3, the foundational concepts of general probability theory are described in the language of measure theory. This mode of thinking will have been mentioned in Chapter 1 with regards to the special cases, and to move on to the general theory, it is extremely easy if one has the knowledge of measure theory.

Chapter 4 and onward are on stochastic processes, which are the central problem of modern probability theory. Although it is impossible to cover the totality of this theory due to limited pages, at least its basic aspects are meant to be comprehensively discussed.

Notes

1. Notations of sets.

$\boldsymbol{R}$ or $\boldsymbol{R}^1$ is the set of all real numbers,

$\boldsymbol{Q}$ is the set of all rational numbers,

$\boldsymbol{N}$ is the set of all natural numbers,

The union of sets is denoted by $\cup$, and in particular, direct sum is represent by + or Σ,

The difference of sets is denoted by \, and in particular, the eigen difference (When $A \supset B$, $A \backslash B$) is denoted by $-$ (i.e. $A - B$).

[1] Translator note: Itô refers to “Iwanami Koza (Lecture Course): Foundation Mathematics”.

2. “Almost every way with respect to measure μ, if

$$f_n \longrightarrow f\,, \qquad |f_n| \leqq g \qquad \left(\int_S g\,d\mu < \infty\right),$$

then

$$\int_S f\,d\mu = \lim_{n\to\infty} f_n\,d\mu."$$

This theorem is called dominated convergence theorem. As there is no appropriate translation for it, it will be translated as **yukai shusoku teiri**. This translated phrase is employed to mean bounded convergence theorem (in the above theorem, a case of $\mu(S) < \infty$, $g =$ constant number). That said, the need of using this special case in a coercively taxing fashion will not occur.

3. In many cases, the hints to exercise questions constitute the outlined proofs. Though there are probably many other ways to solve the problems, by following the hints, I think it will become the practice of applying various theorems discussed in those sections.

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